HYBRID MESONS AND RELATIVISTIC STRINGS

THEODORE J. ALLEN* and M.G. OLSSON
Physics Department, University of Wisconsin-Madison
1150 University Avenue, Madison, Wisconsin, USA

and

SINIŠA VESELI
Theory Group, Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, IL 60510

ABSTRACT

We use the relativistic string to model a QCD flux tube with fixed ends. We demonstrate that the energy of the string well approximates the lattice energies of states with the same quantum numbers recently found by Morningstar et al.

Increasingly sophisticated lattice QCD simulations provide strong evidence that QCD predicts the formation of flux tubes between quarks and antiquarks that are separated by more than a fraction of a fermi. In this talk we will present a relativistic formulation of the effective dynamics of these flux tubes.

The leading order relativistic action for the center of the flux tube should simply be the string tension times the area swept out by the world-sheet of the string.

Hybrid mesons are quark-antiquark boundstates in which the glue is in an excited state. These states are represented by the excited states of the flux tube. The first description of these states by Isgur and Paton[1] was essentially non-relativistic. More recently, Chris Michael[2] showed that his lattice simulations were consistent with the energy formula for a fixed relativistic string.

Lattice QCD allows us to ignore the quark contributions to the action and examine the dynamics of the glue alone. We give a short explanation of the string theory describing the dynamics and quantum numbers of a flux tube with fixed ends.

We start with the Nambu-Goto-Polyakov action, which is the string tension a times the area of the string world-sheet, with fixed end boundary conditions,

$$S_{\rm NGP} \;=\; -\frac{a}{2}\int d\tau \int_0^\pi d\sigma\, \sqrt{-h}\, h^{ab}\partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}, \quad \mathbf{X}(0,\tau) \;=\; \mathbf{x}_1, \quad \mathbf{X}(\pi,\tau) \;=\; \mathbf{x}_2.$$

Classically, this action is equivalent to the square root Nambu-Goto action. The equations of motion that follow from the action are

$$\Box X^\mu = \partial_a(\sqrt{-h}h^{ab}\partial_b X^\mu) = 0, \qquad \partial_a X^\mu \partial_b X_\mu = \frac{1}{2} h_{ab}\, h^{cd}\partial_c X^\mu \partial_d X_\mu. \tag{1}$$

*Presenter

There is a reparametrization invariance of the theory that must be gauge fixed. We choose the orthonormal gauge condition, $\sqrt{-h}\,h^{ab} = \eta^{ab}$, and use the residual symmetry of the action to choose our time coordinate τ in terms of the time in the rest frame of the glue, $X^0 = \frac{P^0}{\pi a}\tau$. We may solve the equations of motion (1) by making a mode expansion of the string position.

$$\mathbf{X}(\sigma,\tau) \;=\; \frac{\sigma}{\pi}\mathbf{R} - \frac{1}{\sqrt{\pi a}}\sum_{n\neq 0}\frac{\hat{\alpha}^k_n}{n}\sin(n\sigma)e^{-in\tau}, \tag{2}$$

where $\mathbf{R} = \mathbf{x}_2 - \mathbf{x}_1$. Upon quantization, the Fourier coefficients $\hat{\alpha}^i_n$ become independent oscillators obeying the commutation relations $[\hat{\alpha}^i_n, \hat{\alpha}^j_m] = m\,\delta_{n+m,0}\,\delta^{ij}$. The second equation of motion in (1) may be expressed in terms of its Fourier coefficients,

$$L_n \;=\; \sum_m \hat{\alpha}_m\cdot\hat{\alpha}_{n-m} = 0, \quad \text{with} \quad \hat{\alpha}_0 \equiv -\mathbf{R}\sqrt{\frac{a}{\pi}}. \tag{3}$$

This equation of motion is actually a constraint in the Dirac sense and will be used to define physical states. We use the Gupta-Bleuler method, in which we require that the "annihilation" parts of the constraints annihilate the physical states, to implement the constraints. The constraint conditions $L_n = 0$ are in fact second class because even classically they do not form a closed algebra.

We select the physical states from the Fock space of the oscillators. The states of the Fock space are fully described by the number of phonons in each of the modes. Thus we will have $\mathbf{N}_1, \mathbf{N}_2, \ldots$ phonons in the $n = 1, 2, \ldots$ modes, etc.

$$\begin{aligned} \text{Fock space} \;&=\; |\mathbf{R}, \mathbf{N}_1, \mathbf{N}_2, \ldots\rangle, \\ L_{-n}\,|\psi_{\text{physical}}\rangle \;&=\; 0, \quad n \geq 0. \end{aligned} \tag{4}$$

The Virasoro conditions (4) for non-zero n eliminate the "longitudinal" modes. To see this, we imagine taking $R \to \infty$ and ignoring the piece quadratic in the oscillators.

$$L_n \;\sim\; R\,\hat{\alpha}^L_n \equiv R\,(\hat{\mathbf{R}}\cdot\hat{\alpha}_n) \tag{5}$$

For ease in describing the physical states, we will assume that we can treat them as strictly transverse for any fixed separation R. The same analysis can be applied to the exact states with more work, yielding the same results. It is convenient to work in the helicity basis for the transverse phonons.

The mass-shell relation $(P^0)^2 = (aR)^2 + 2\pi a(N + c)$ follows from the zero-mode Virasoro condition.

$$\begin{aligned} L_0 \;&=\; \frac{1}{2\pi a}\left\{-(P^0)^2 + (aR)^2 + 2\pi a(N+c)\right\}, \\ N \;&=\; \sum_{n=0}^{\infty}\hat{\alpha}_{-n}\cdot\hat{\alpha}_n \;=\; \sum_{n=1}^{\infty} n(N^+_n + N^-_n), \end{aligned} \tag{6}$$

where N^+_n and N^-_n are respectively the number of phonons in the mode n with positive and negative helicity.

The first quantum number we examine is the angular momentum, Λ, of the glue along the quark axis. We find

$$\Lambda \;=\; \hat{\mathbf{R}} \cdot \mathbf{L} \;=\; \frac{i}{2}\epsilon_{ijk}\hat{R}^i \sum_{n=1}^{\infty} \frac{1}{n}\alpha_n^{[j}\alpha_{-n}^{k]} \;=\; \sum_{n=1}^{\infty}(N_n^+ - N_n^-). \tag{7}$$

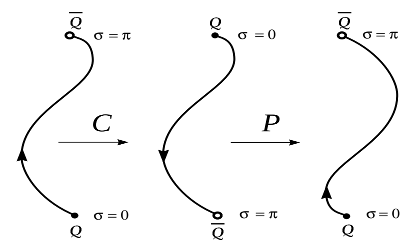
The second quantum number of the glue is its CP value. Charge conjugation turns a quark into an antiquark and reverses the flux. The effect is to take $\sigma \to \pi - \sigma$. Parity then simply takes a position vector to its negative. We find

$$\mathbf{X}(\sigma) \overset{CP}{\to} -\mathbf{X}(\pi - \sigma) \quad \Rightarrow \quad \alpha_n \overset{CP}{\to} (-1)^n \alpha_n \quad \Rightarrow \quad CP = (-1)^N. \tag{8}$$

The last quantum number is the parity under reflection in a plane containing the quark axis. This is a symmetry of the system and implies a degeneracy in the $+\Lambda$ and $-\Lambda$ states. For states with $\Lambda = 0$, the states may be classified by their parity under this reflection.

We use the standard molecular notation to label the various string excitation states. States with $\Lambda = 0, 1, 2, \ldots$ are denoted by $\Sigma, \Pi, \Delta, \ldots$, which is just a fancy way of saying $S, P, D, \ldots$. The state's value of CP is denoted by a subscript g or u depending on whether it is even or odd. Finally, the Σ states are superscripted with the value of the reflection parity through a plane containing the quark axis.

Fig. 1. The action of CP.



Colin Morningstar graciously sent us the results[3] of one of his lattice simulations for the energy of two fixed quarks. We analyzed these data by fitting the lowest state with $N = 0$ string energy plus a one gluon exchange potential,

$$V_{\Sigma_g^+} \;=\; -\frac{4\alpha_s}{3r} + \sqrt{(ar)^2 + 2\pi ac} + C. \tag{9}$$

to obtain the best fit values

$$\begin{aligned} \alpha_s &= \;\; 0.234, & ar_0^2 &= 1.34, \\ Cr_0 &= -2.51, & c &= 0. \end{aligned}$$

We then compare the data to the now parameter-free curves

$$\begin{aligned} V &= \sqrt{(ar)^2 + 2\pi a(N + c)} + C \quad \text{(solid lines)}, \\ V &= \frac{\alpha_s}{6r} + \sqrt{(ar)^2 + 2\pi a(N + c)} + C \quad \text{(dashed lines)}. \end{aligned} \tag{10}$$

The first potential is just a shifted string energy (lattice data do not determine absolute energies) while the second has an octet one gluon exchange potential added in. The argument for this is that as $r \to 0$ the string may continue on to a single constituent gluon plus quark-antiquark state, in which case the quarks are in a color octet state. It is important to note that these are not "fits," but are "predictions" as we have determined all the parameters from the lowest energy Σ_g^+ state. The data either fit or do not fit. Notice that there is general agreement for the states Σ^+, Π, and Δ, with the Π states being especially striking. On the other hand, the Σ^- states do not agree well.

We have shown by construction that single relativistic strings can be consistently constructed in $D = 4$ to model flux tubes. Though this theory is only an effective theory, it can be pushed quite far. We found that recent lattice data were reasonably well predicted by relativistic string models, except for the Σ^- states. The Σ^- states are a puzzle in this formalism, though Chris Michael reminds us in his plenary talk that Σ_u and Π_u must be degenerate at $r = 0$.

Fig. 2. The Σ states. ($\Lambda = 0$)

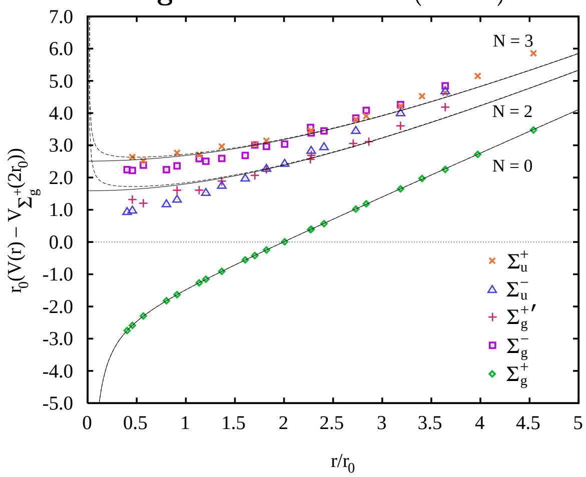


Fig. 3. The Π states. ($\Lambda = 1$)

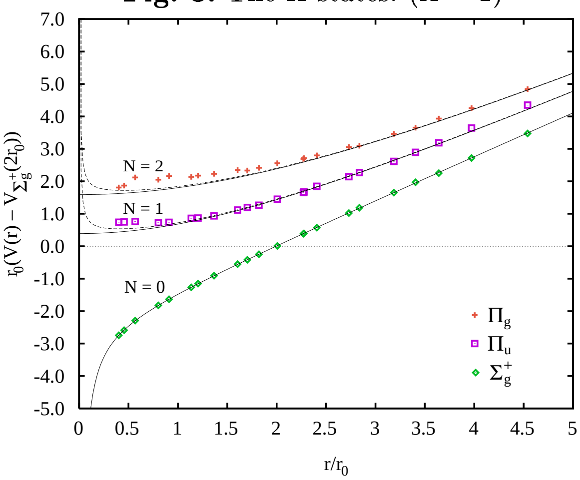
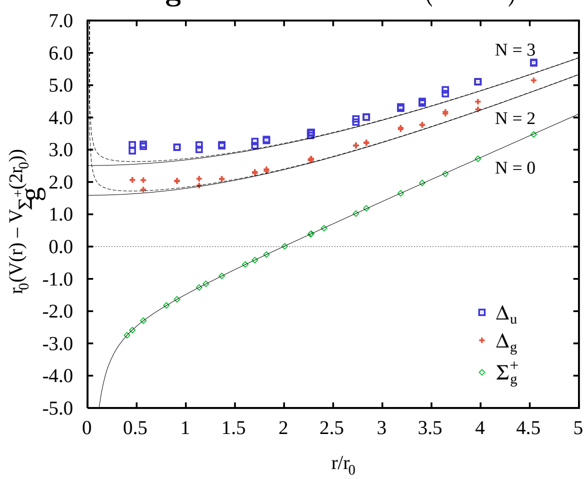


Fig. 4. The Δ states. ($\Lambda = 2$)



References

1. N. Isgur and J. Paton, *Phys. Rev.* **D31** (1985) 2910.
2. S. Perantonis and C. Michael, *Nucl. Phys.* **B347** (1990) 854.
3. K.J. Juge, J. Kuti, and C.J. Morningstar, Proceedings of the XVth International Symposium on Lattice Field Theory, *Nucl. Phys. B* (Proc. Suppl.) **63** (1998) 326; hep-lat/9709131.